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PROPELLER BLADE STRESSES CAUSED BY PERIODIC DISPLACEMENT OF THE PROPELLER SHAFT

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SUMMARY

The present reports deal with different vibration stresses of the propeller and their removal by an elastic coupling of propeller and engine. A method is described for protecting the propeller from unstable oscillations and herewith from the thus-excited alternating gyroscopic moments. The respective vibration equations are set down and the amount of elasticity required is deduced.

INTRODUCTION

One effective means of safeguarding the propeller against alternating bending moments is the elastic union of engine and propeller. However, such a design, satisfactory under any and all conditions, does not exist at the present time, because of the constructional difficulties involved. Propeller blades hinged from all sides in the manner of helicopter rotor blades would be an ideal solution (reference 1). But because of the high loading imposed on the hinges this method is not at once practicable on normal blades.

To reduce, in particular, the coupling between propeller bending stresses and crankshaft torsional vibrations the flexible torque drive has been developed as a partly elastic combination. And, since the torsional vibrations

*"Die Beanspruchung der Luftschraubenflügel bei periodischer Verlagerung der Schraubenwelle." Luftfahrtforschung, vol. 18, no. 11, Nov. 20, 1941, pp. 383-386.

of the crankshafts themselves have been successfully overcome by dampers and centrifugal pendulums the danger of overstressing a propeller by blade vibrations which should hold a crankshaft torque in balance, may be regarded as non-existing in many cases.

Lately, however, there have been cases of propeller failure which were obviously caused by pitching motions of the whole engine. The motion of the propeller, as a whole, can be divided in a displacement within the plane of the propeller disk and in an unstable vibration about the center of gravity of the propeller, while the former produces mass forces and moments within the plane of the swept disk, the latter creates alternating gyroscopic moments at right angles to the former so that both combined may induce natural bending vibrations in the blades. These phenomena have been studied extensively in the U. S. (reference 5). The excitation of these vibrations is due to gas and mass forces on the engine side or to air loads, as, for instance, when the blades miss in striking past flow obstacles, on the propeller side. These two moments are analyzed in the present paper.

II. STRESS DUE TO VIBRATION OF THE PROPELLER SHAFT ABOUT AN AXIS AT RIGHT ANGLES TO IT

In the following:

ωt angle of rotation of the propeller blade about the axis of rotation

φ angle of rotation of the propeller shaft about an axis at right angles to it

m blade mass

s distance of the center of gravity from the blade root

$J = \int_{r_1}^{R} r^2\, dm - r_1 \int_{r_1}^{R} r\, dm$ with r_1 as reference radius for the blade root

For the calculation, it is recommended to consider a mass element of the propeller blade (fig. 1), the speed v of which is computed from the rotation speeds ω and $\dot{\varphi}$. Differentiation with respect to time then affords the acceleration and hence the force of inertia of the mass element, which, after integration along the whole blade gives the bending moment on the blade root.

The acceleration at right angles to the plane of the propeller disk is divided in two parts

$$\frac{dv}{dt} = \frac{dv_1}{dt} + \frac{dv_2}{dt}$$

where

$$v_1 = r \sin \omega t \, \dot{\varphi}; \quad v_2 = r \cos \omega t \, \omega$$

whence

$$\frac{d v_1}{dt} = r \left(\omega \dot{\varphi} \cos \omega t + \ddot{\varphi} \sin \omega t\right)$$

$$\frac{d v_2}{dt} = r \cos \omega t \, \omega \dot{\varphi}; \text{ because } \frac{d \overline{v}_2}{dt} = v_2 \frac{d \overline{\varphi}}{dt}$$

Herewith the bending moment follows at

$$M\varphi = J \left(\ddot{\varphi} \sin \omega t + 2 \omega \dot{\varphi} \cos \omega t\right).$$

φ may be written $\varphi = \Phi \sin \gamma t$, wherewith the preceding expression takes the form

$$M\varphi = - J\gamma^2 \Phi \left\{\sin \gamma t \sin \omega t - 2 \frac{\omega}{\gamma} \cos \gamma t \cos \omega t\right\}$$

$$= \frac{J}{2} \gamma^2 \Phi \left\{\left[1 + 2 \frac{\omega}{\gamma}\right] \cos (\gamma + \omega) t - \left[1 - 2 \frac{\omega}{\gamma}\right] \cos (\gamma - \omega) t\right\}$$

For $\gamma = \omega$ we get

$$M\,\varphi = \frac{J}{2}\,\gamma^2\,\Phi\left\{3\cos 2\,\omega\,t + 1\right\}$$

On this bending moment, at right angles to the plane of the propeller disk, is superposed a moment due to the mass forces which is released by the center of gravity motion of the propeller. The amplitude of this vibration is given by $A = -p\,\Phi$ where p is the distance up to the rotation polar; hence we can put $x = A \sin \gamma t$. Then the bending moment in the plane of the propeller disk becomes

$$M_x = -m\,s\,\gamma^2\,A\sin\gamma\,t\cos\omega\,t$$

$$= -\frac{m}{2}\,s\,\gamma^2\,A\left\{\sin(\gamma+\omega)\,t + \sin(\gamma-\omega)\,t\right\}$$

If $\gamma = \omega$ then

$$M_x = -\frac{m}{2}\,s\,\gamma^2\,A\sin 2\,\omega\,t$$

III. STRESS DUE TO VIBRATIONS OF PROPELLER SHAFT ABOUT TWO AXES AT RIGHT ANGLES TO IT

If, in addition, the propeller shaft vibrates about the second axis perpendicular to it with the deflection

$$\psi = \Psi\cos\gamma\,t, \quad B = p\,\Psi, \quad y = B\cos\gamma\,t$$

so that the propeller center describes an ellipse rather than a straight line, the bending moment at right angles to the plane of the propeller disk becomes

$$M\varphi,\psi = J\left\{\ddot{\varphi}\sin\omega\,t + 2\,\omega\,\dot{\varphi}\cos\omega\,t + 2\,\omega\,\dot{\psi}\sin\omega\,t - \ddot{\psi}\cos\omega\,t\right\}$$

$$M\varphi,\psi = J \left\{ -\gamma^2 [\Phi \sin \gamma t \sin \omega t - \Psi \cos \gamma t \cos \omega t] + 2 \omega \gamma [\Phi \cos \gamma t \cos \omega t - \Psi \sin \gamma t \sin \omega t] \right\}$$

$$M\varphi,\psi = J \left\{ \frac{\Phi + \Psi}{2} [\gamma^2 + 2 \omega \gamma] \cos (\gamma + \omega) t - \frac{\Phi - \Psi}{2} [\gamma^2 - 2 \omega \gamma] \cos (\gamma - \omega) t \right\}$$

and the respective mass moment

$$M_{x,y} = m s \left\{ \ddot{x} \cos \omega t - \ddot{y} \sin \omega t \right\}$$

$$M_{x,y} = -m s \gamma^2 \left\{ A \sin \gamma t \cos \omega t - B \cos \gamma t \sin \omega t \right\}$$

$$M_{x,y} = -m s \gamma^2 \left\{ \frac{A - B}{2} \sin (\gamma + \omega) t + \frac{A + B}{2} \sin (\gamma - \omega) t \right\}$$

In the specific case of circular vibration, that is, $\Phi = \Psi$, $A = -B$ (precession in reverse rotation) we get

$$M\varphi,\psi = J \gamma^2 \Phi \left\{ \left[1 + 2 \frac{\omega}{\gamma} \right] \cos (\gamma + \omega) t \right\}$$

$$M_{x,y} = - m s \gamma^2 A \sin (\gamma + \omega) t$$

by precession at constant speed, that is, $\Phi = -\Psi$ and $A = +B$,

$$M\varphi,\psi = - J \gamma^2 \Phi \left\{ \left[1 - 2 \frac{\omega}{\gamma} \right] \cos (\gamma - \alpha) t \right\}$$

$$M_{x,y} = - m s \gamma^2 A \sin (\gamma - \omega) t$$

Accordingly the maximum values of the alternating bending moments are exactly twice as high by the circular vibration as by one degree of freedom.

Now, so long as the propeller is carried from the shaft no all-around elastic attachment of the propeller is possible, as it would sag under its own weight. Therefore the displacement prescribed by the propeller shaft is always shared by the propeller itself. But one method of mounting should be conceivable by which the propeller would at least not be forced to join the unstable oscillation in its full extent. As is apparent from the foregoing, the two alternating moments in both instances differ in phase by 90°, so the moment vector in the blade root makes one revolution of 360° during one vibration period. Since the fatigue strength, at least of isotropic materials, depends solely upon the magnitude of the alternating moments but not on whether they act along one or all directions of the cross section, the elimination of the gyroscopic moments would be of service only if they were substantially greater than the mass moments. For comparison the blade moment of inertia is, because of its practically linear mass distribution, expressed with $J = m \; l^2/6$, where l = length of blade, and $s = l/3$, whence in the most unfavorable case

$$M_{\varphi,\psi} = J \, \gamma^2 \, \Phi \left(1 + 2 \frac{\omega}{\gamma}\right) = - \frac{m}{6} \, l^2 \gamma^2 \, \frac{A}{p} \left(1 + 2 \frac{\omega}{\gamma}\right)$$

$$M_{x,y} = - m \, s \, \gamma^2 \, A = - \frac{m}{3} \, l \, \gamma^2 \, A$$

$$\frac{M_{\varphi,\psi}}{M_{x,y}} = \frac{\left(1 + 2 \frac{\omega}{\gamma}\right) l}{2 \, p}$$

This ratio becomes greater than 1 for larger values of γ, if radial engines with large propellers are involved, as then l is great relative to p. In the rather frequent case that the vibration is excited by an unbalance of the propeller, or $\gamma = \omega$, this ratio can become very great;

$$\frac{M_{\varphi,\psi}}{M_{y,x}} = \frac{1.5 \, l}{p} \sim 4$$

Still other reasons speak meanwhile for a removal of the gyroscopic moments. Added up over all blades they combine, in their effect on the propeller shaft and try to strain it, against which the mass moments mutually cancel and only press the propeller shaft more or less severly against its bearings. Lastly this type of stress appears to be detrimental for wooden blades respecting their attachment in the steel sleeve, which, however, would first have to be proved by experiments.

It is to be noted that the previously computed bending moments by given propeller displacement Φ, Ψ and A, B, represent minimum values, hence may actually become a multiple of it by increasing resonance.

It is, therefore, apparent that a supplementary shock absorber that protects the propeller from alternating gyroscopic moments has its advantages, so far as this may be accomplished, with a minimum increase in weight.

IV. ANALYSIS OF A SUPPLEMENTARY SHOCK-ABSORPTION SYSTEM BETWEEN ENGINE AND PROPELLER

An analysis, such as this, must proceed from the principle that no new critical speeds are created in the operating speed range and that the gyroscopic moment induced in curve flight does not tilt the propeller in excess of 10° relative to the airplane.

An attempt is made to analyze both requirements.

Figure 2 is a diagrammatic view of a vibration system formed by elastic suspension.

θ_1, θ_2 moments of inertia of engine plus propeller mass about the normal and the lateral axis, respectively

c_1 elasticity of engine about the normal axis

c_2 elasticity of engine about the lateral axis

C elasticity of hub (about all axes)

J_p, J the respective polar and equatorial moment of inertia of the propeller

ω angular velocity of propeller

$\overline{\omega}$ angular velocity of airplane in banking

γ angular deflection of engine and propeller about the lateral axis

φ, α angular deflection of engine and propeller about the normal axis

ψ, β angular deflection of engine and propeller about the lateral axis

Then the motion equations of the vibration system shown in figure 1 read:

$$\left.\begin{array}{ll} \text{I} & \Theta_1 \ddot{\varphi} + c_1 \varphi + C (\varphi - \alpha) = 0 \\ \text{II} & \Theta_2 \ddot{\psi} + c_2 \psi + C (\psi - \beta) = 0 \\ \text{III} & J \ddot{\alpha} + C (\alpha - \varphi) - J_p \omega \dot{\beta} = 0 \\ \text{IV} & J \ddot{\beta} + C (\beta - \psi) + J_p \overline{\omega} \dot{\alpha} = 0 \end{array}\right\} \quad (1)$$

which, with

$$\varphi = \Phi \sin \gamma t \qquad \alpha = A \sin \gamma t$$

$$\psi = \Psi \cos \gamma t \qquad \beta = B \cos \gamma t$$

gives

$$\left.\begin{array}{ll} \text{I} & (c_1 + C - \Theta_1 \gamma^2) \Phi = C A \\ \text{II} & (c_2 + C - \Theta_2 \gamma^2) \Psi = C B \\ \text{III} & (C - J \gamma^2) A + J_p \omega \gamma B = C \Phi \\ \text{IV} & (C - J \gamma^2) B + J_p \omega \gamma A = C \Psi \end{array}\right\} \quad (2)$$

Multiplying equations I and II in (2) by C and entering III and IV give

$$\left.\begin{array}{l} \left\{c_1 + C - \Theta_1 \gamma^2\right\} \left\{(C - J \gamma^2) A + J_p \omega \gamma B\right\} = C^2 A \\ \left\{c_2 + C - \Theta_2 \gamma^2\right\} \left\{(C - J \gamma^2) B + J_p \omega \gamma A\right\} = C^2 B \end{array}\right\} (3)$$

The four natural frequencies then follow from the equation of the eighth order for γ, left after the elimination of A and B from (3). For simplicity we put $c_1 = c_2 = c$ and $\Theta_1 = \Theta_2 = \Theta$ which is approximately correct for radial engines, whence (3) gives

$$\gamma^4 - \gamma^2 \left\{\frac{C}{J} + \frac{c + C}{\Theta}\right\} + \frac{c\ C}{J\ \Theta} \pm 2\,\omega\,\gamma\ \left(\gamma^2 - \frac{c + C}{\Theta}\right) = 0 \qquad (4)$$

because $A = + B$. Herefrom the four natural frequencies are computed from the position of which quantity C is defined. They should be located so high that the corresponding critical revolutions per minute come far below the range of service revolutions per minute. However, equation (4) need not be resolved rigorously, because C should at least be flexible enough so that the propeller scarcely joins in the pitching motions of the engine, that is, the coupling between I, II, III and IV in (1) is very loose. The coupling factor is

$$K = \sqrt{\frac{C}{c + C}}$$

as on a two-mass system which in this instance must be of the order of magnitude of < 0.3. But in that event the incorrect coupling frequencies are not much different from the natural frequencies derived from the uncoupled equations. They are obtained from (1) by putting α and β in I and II and φ and ψ in III and IV = 0. Then equation (4) can be written for the simplified case that the uncoupled pitching motions of the engine about the normal and lateral axis are equivalent, as follows:

$$\left\{\gamma^2 - \frac{c + C}{\Theta}\right\} \left\{\gamma^2 \pm 2\,\omega\,\gamma - \frac{C}{J}\right\} = 0 \qquad (5)$$

Herewith the natural vibrations become

$$\gamma_{1,2} = \sqrt{\frac{c + C}{\Theta}} \qquad \gamma_{3,4} = \sqrt{\frac{C}{J}}\ \sqrt{\frac{v}{v \pm 2}}$$

with $v = \gamma/\omega$. If k_L is the mode of excitation on the propeller side and k_M on the engine side, we get $v = k_L$ in the first instance, and $v = k_M\, i$ in the second, with i as reduction gear ratio. Hence, the greater v is, the less γ_3 and γ_4 differ, from each other.

This brings us to quantity C.

Let ϖ_{max} indicate the maximum angular velocity of the airplane in banking and M_k the gyroscopic moment. In order that the propeller even at ω_{max} does not incline more than 10° from its mormal position, it must

$$C = \frac{M_{k\ max}}{0.175} \tag{6}$$

whereby

$$M_{k\ max} = J_p\, \omega_{max}\, \varpi_{max}$$

wherewith the two natural frequencies of the propeller are:

$$\gamma_{3,4} = 3.4 \sqrt{\omega_{max}\, \varpi_{max}} \sqrt{\frac{v}{v \pm 2}}$$

The thus-defined natural frequencies are therefore dependent upon the size of the propeller itself only to the extent that the value ω_{max} is defined by it. To illustrate: take a 1500 horsepowered engine with $i = 1.6$ and $\omega_{max} = 150\ s^{-1}$, for which in figure 2 the two natural frequencies n_3 and n_4 min^{-1} are plotted against v for a value of $\varpi_{max} = 2\ s^{-1}$. The natural frequencies n_m of the elastically suspended engine themselves will be situated in the neighborhood of $n_m = 1000\ min^{-1}$, so that all critical speeds are well below the speed range of operation.

The lowest mode of excitation would be $k_L = 3$ on the propeller side and $k_M = 1.5$ on the engine side, since v must be greater than 2 if the higher of the two natural frequencies of the propeller is to be able to be excited at all. According to figure 3 the higher natural frequency for $v = 1.5 \times 1.6 = 2.4$ is amply high. If the conditions are such that this mode of excitation must be reckoned with, quantity C should be lowered. But, as before, the propeller would already incline more than 10° in banking at $\overline{\omega}_{max} < 2^{-1}$, stops to prevent this would be necessary. For excitations of > 3 on the propeller side and > 2 on the engine side all critical speeds would remain well below the operating speed range even by more severe springing C.

In conclusion a brief check of quantity C is made in view of the coupling factor formed by it.

According to equation (6) we get a value of

$$C = 685{,}000 \text{ cm kg}$$

with $J_p = 400$ kg cm², $\omega_{max} = 150\ s^{-1}$ and $\overline{\omega}_{max} = 2\ s^{-1}$

If n_M is to be $n_M = 1000\ \text{min}^{-1}$, that is,

$$\nu_M = \sqrt{\frac{c}{\Theta}} = 105\ s^{-1}$$

it follows with $\Theta = 1300$ cm kgs² and $c = 1300 \times 11000 = 14{,}300{,}000$ cm kg that

$$K = \sqrt{\frac{C}{c + C}} = 0.214$$

Herewith the uncoupling of engine and propeller pitching vibration is insured and the correctness of the approximation of the natural vibrations proved.

Translation by J. Vanier,
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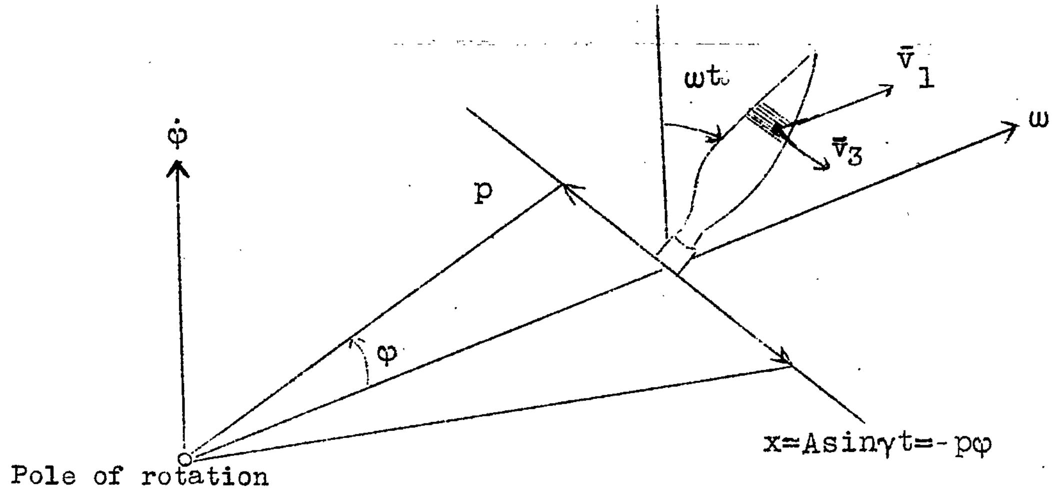

Figure 1.- Derivation of inertia forces produced on the blade due to propeller shaft displacement.

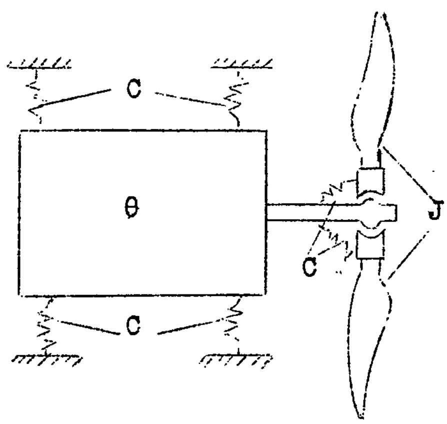

Figure 2.- Diagrammatic view of vibration system of engine with elastically mounted propeller.

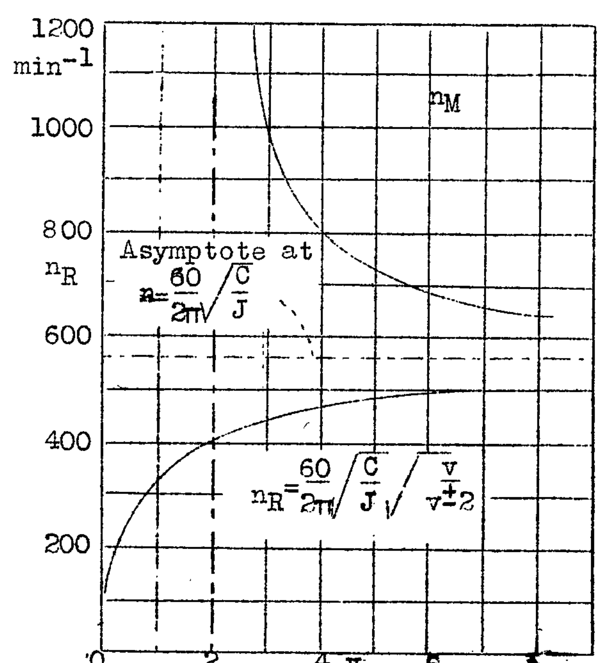

Figure 3.- Resonant frequencies n_R of the elastically mounted propeller for the various modes of excitation $v=k_L$ and $i \cdot k_M$.